Pricing Models for Non-Agency RMBS: Approaches and Model Risks

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Agenda

• An introduction to non-agency RMBS securities
• Approaches of mortgage pricing: structural vs. reduced-form
• Modeling tranche cash flows
• Assessing tranche values from cash-flow estimates
• Model validation and model risk
A refresher on non-agency RMBS securities

- Tranches of a static pool of mortgage loans
  - Subprime
  - Alt-A
  - Option ARM
  - Prime jumbo

- Senior/subordinate structure to create credit enhancement
  - Losses directed to subordination first
  - Other forms of CE: excess spreads; overcollateralization; MI

- Waterfall rules on allocation of principal and interest payments across senior and subordination tranches
  - Shifting principal payments toward senior tranches
  - Triggers based on credit performance of collateral
An simple illustration

Underlying Collateral: mortgage loans

Interest payments

Principal payments (scheduled, prepayment, recovery)

Waterfall rules

Senior tranches

Subordination tranches
Subprime ABX AAA prices

Source: BofA Merrill Lynch Global Research, Markit
Approaches to mortgage pricing

- Risks of mortgage
  - Prepayment risk
  - Default risk
  - Spread risk

- Main Drivers
  - Interest rate
  - Housing price
Structural approach (option-pricing models following Black-Scholes-Merton model, e.g., Kau and Keenan 1995)

- Prepayment: American call options on the mortgage
- Default: American put options on the underlying asset
- Endogenous decisions on prepayment and default as efficient execution of options
Structural approach (option-pricing models) – a quick summary

- The value of a mortgage is $V(t, R(t), H(t))$
  - $R(t)$ – interest rate process
  - $H(t)$ – house price process
  - Both following SDEs

- Assuming that $H(t)$ is a tradable asset, and derive the fundamental PDE for the mortgage value under no arbitrage
  - The boundary conditions are given by the optimal decision rules on exercising the prepayment and default options
Approaches to mortgage pricing

Difficulties in applying the structural approach:

- Borrowers not executing options efficiently
- Housing market is not efficient
- Not able to use forward simulation as decisions are endogenous
- Backward solution often computationally intensive (especially in the presence of path dependency)
- Not conducive for estimation from empirical data
Reduced form pricing model:

- Prepayment and default behavior estimated from empirical data
- Following the doubly stochastic default model framework (Lando 1998 and Duffie and Singleton 1999)
- Given prepayment and default behavior, mortgage value can be obtained using forward pricing
- This is the approach commonly adopted in practice
Doubly stochastic default:

τ: time of default

λ(t): default density at t

- λ(t)dt is the probability of default between t and t+dt: Prob{τ≤t+dt| τ>t}, or
- the rate of decline of survival probability: dS(t) = - λ(t)S(t)dt

λ(t) is stochastic, conditional on a path of λ(t) the conditional survival probability is

\[ \text{Pr} \{ \tau > t | \lambda(u) : 0 \leq u < t \} = \exp[-\int_0^t \lambda(u)du] \]

The unconditional survival probability is obtained by simply taking expectation over λ(t):

\[ \text{Pr} \{ \tau > t \} = E \exp[-\int_0^t \lambda(u)du] \]
Doubly stochastic default:

In the case of mortgage:

\( \lambda(t) \) is simply \( MDR(t) \), often assumed to be constant between \( t \) and \( t+1 \)

MDR(t) is stochastic, modeled as driven by interest rate and house price

For a given interest rate and house price path, the conditional survival probability is simply

\[
S(t \mid R(u), H(u) : 0 \leq u < t) = \prod_{i=0}^{t-1} (1 - MDR_i)
\]

The unconditional probability of survival can be obtained by taking average over the interest rate and house price paths
Reduced form pricing model:

Under the doubly stochastic default framework, consider a zero coupon bond paying $1 at time $T$:

Risk-free bond price is

$$\delta(t,T) = E_t^Q[\exp(-\int_t^T R(u)du)]$$

Bond price with default risk (assuming zero recovery):

$$\delta^d(t,T) = E_t^Q[\exp(-\int_t^T R(u)du) \cdot I(\tau > T)] = E_t^Q[\exp(-\int_t^T R(u) + \lambda^*(u)du)]$$
Approaches to mortgage pricing

Reduced form pricing model:

Application of this approach to mortgage valuation (e.g., Kau, Keenan, and Smurov 2006, and Liao, Tsai, and Chiang 2008)

Competing risks: prepayment and default hazard

\[ \lambda(t) = \lambda_p(t) + \lambda_d(t) \]

Theoretically, a high prepayment hazard is associated with a low default hazard and vice versa.

Mortgage value = Value of prepayments + Values of recovery from defaults + Value of scheduled payments if no prepayment or default
Approaches to mortgage pricing

Reduced form pricing model:

\[ V(t) = E_t^Q \left[ \int_t^T M(u) \exp\left[ -\int_t^u \left( R(s) + \lambda_p^*(s) + \lambda_d^*(s) \right) ds \right] \lambda_p^*(u) du \right] \]

\[ + E_t^Q \left[ \int_t^T w(u)M(u) \exp\left[ -\int_t^u \left( R(s) + \lambda_p^*(s) + \lambda_d^*(s) \right) ds \right] \lambda_d^*(u) du \right] \]

\[ + E_t^Q \left[ \int_t^T Y(u) \exp\left[ -\int_t^u \left( R(s) + \lambda_p^*(s) + \lambda_d^*(s) \right) ds \right] du \right] \]

M(t): scheduled UPB

w(t): recovery ratio

Y(t): scheduled principal and interest payment
Modeling tranche cash flows

Risk drivers:

• Interest rate paths: using term structure models

• House price paths: modeling housing price diffusion

Collateral cash flows:

• Scheduled payments

• Prepayment at time $t$,

\[ \prod_{i=1}^{t-1} (1 - \text{SMM}_i - \text{MDR}_i) \cdot \text{SMM}_t \cdot \text{ScheduledUPB}_t \]

• Default at time $t$

\[ \prod_{i=1}^{t-1} (1 - \text{SMM}_i - \text{MDR}_i) \cdot \text{MDR}_t \cdot (1 - \text{Severity}_t) \cdot \text{UPB}_{t-1} \]
Modeling tranche cash flows

Tranche cash flows:

- Waterfall structure (Intex)
- Servicer advances

P&I advance rate by delinquency status

Source: Loan Performance and Freddie Mac

- MI haircuts
Assessing tranche values from cash-flow estimates

- Risk-neutral versus actual default probability
  - The difference reflect the risk premium on default risk
  - In mortgage pricing, the risk premium is typically captured by an optional-adjusted spread (OAS): A constant spread over the reference curve
  - Trading volume is thin – very little information on market prices
  - Pricing often based on capital cost: potential large variations
Model validation and model risk

Model validation

- Risk-based approach
- Identify model risks associated with model use
- Assess existing mitigating factors and design validation work for each model risk
- Key challenges:
  - Lack of pricing data
  - Regime change post-housing crisis
  - Foreclosure timeline extensions
  - Modifications
  - Strategic defaults
Model validation

- Core models (e.g., prepayment, default, and severity)
  - Model theory
  - Functional form
  - Variable selection
  - Sample selection
  - Estimation/calibration methods

- Model use (e.g., business transactions, financial reporting)
  - Model dependency
  - Data integrity
  - System implementation
Default model: potential bias due to foreclosure timeline extension and modifications

A hypothetical example of default prediction
Model validation and model risk

Default model validation

• Evaluating key assumptions:
  o The effect on default of foreclosure timeline extension
    ▪ Any effect on curing
  o The effect of modification program
    ▪ Treatment-effect approach: comparing modified loans with unmodified loans
    ▪ Compare performance of modified loans to other delinquency statuses
    ▪ What about effects on ultimate losses?
Default model validation

- Back-testing using adjusted “actual” default rates
  - What would be default rates without foreclosure timeline extension and modifications?
  - Using historical roll rates

- Scenario analysis on default timing
  - Default timing is potentially important for structured products
  - Senior tranches tend to benefit from frontloading defaults and subordination tranches tend to benefit from delay of defaults
Model validation and model risk

Default model validation:

• Benchmarking against alternative models
  o Transition model vs a single-state default model
  o Simple roll rates model

• Benchmarking against dealer models

Subprime cum defaults

<table>
<thead>
<tr>
<th>Vintage</th>
<th>JPM</th>
<th>CITI</th>
<th>BC</th>
<th>BOA</th>
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</table>

Source: JP Morgan, Citi, Barclay, and BoA
Model validation and model risk

Prepayment model: regime change after the housing crisis

<table>
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<th>Product</th>
<th>2010H1</th>
<th>2010H2</th>
<th>Difference</th>
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<td>4.1</td>
<td>0.5</td>
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<tr>
<td>Subprime ARM 2/28</td>
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<td>1.6</td>
<td>0.8</td>
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<tr>
<td>Subprime ARM 3/27</td>
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<td>2.0</td>
<td>0.6</td>
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<tr>
<td>Subprime ARM 5/25</td>
<td>2.9</td>
<td>3.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

*Source: Loan Performance, Freddie Mac*

- Subprime borrowers can prepay when house price rises
  - Credit curing
  - Cash-out refi
- Post-housing crisis, prepayment speeds very low
- Speeds not responsive to rate movements

Model based on historical data no longer applicable: using scenario analysis
Model validation and model risk

Severity model: selection problem from using state-level house price

- Model was estimated using zip-level HPI
- For projection, using state HPI forecasts
- Forecasted severities tend to be lower than recent actual severities
- Why? Selection problem: recent defaults tend to concentrate in zips with worse house price decline
- Solution: quantify the potential bias and make adjustment to model forecasts accordingly
Model validation and model risk

Data integrity: OTS vs MBA definitions of delinquency status

- Intex data: the delinquency status reported by servicers can be either OTS and MBA convention
- OTS: D30 means missing two payments
- MTA: D30 means missing one payment
- Model should be able to distinguish the two conventions
- Solution: determining OTS/MBA convention based on trustee reports and other data sources
Model validation and model risk

System implementation risks:

- Coding errors in model implementation
- Wrong parameter files deployed or not mapped correctly to products
- Model inputs not correct
- Model outputs not correctly consumed by downstream processes
Model validation and model risk

Model governance framework

• Model inventory
  o Model definition
  o Model vs Model Use
  o Model risk rating

• Model development reviews

• On-going model risk assessment: model risk reports
  o Assessment on existing model risks
  o Identification of new and emerging model risks
  o Performance monitoring
    o Threshold design

• Model change reviews

• Model new use reviews